Positioning Analysis: Marketing Engineering Technical Note[1]

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Introduction

This technical note outlines the "rocket science" behind positioning analysis. Formal positioning analysis is based on three core concepts: (1) Customer perceptions, (2) Customer preferences, and (3) Customer choices. Perceptions refer to beliefs that customers have about various offerings available in the markets (e.g., A Volvo is a safe car). Preferences refer to mental states or processes that give customers the ability to set in their minds one offering before another in terms of their overall desirability to them (e.g., I

[1] This technical note is a supplement to the materials in Chapter 4 of Principles of Marketing Engineering, by Gary L. Lilien, Arvind Rangaswamy, and Arnaud De Bruyn (2007). Visit www.decisionpro.biz for additional information.

prefer a BMW to Volvo). Choices involve the process of judging the merits of multiple offerings and selecting one for further consideration or purchase (e.g., I want to test drive a BMW). Choice does not necessarily mean purchase – one could choose online and purchase offline. Generally, we believe that perceptions drive preferences, which in turn, trigger choices and purchases. However, people need not necessarily purchase items they prefer more – for example, many people prefer healthy items to less healthy items, but they are also likely in some cases to choose potato chips over yogurt. Also, in some situations, preferences need not necessarily be driven by perceptions, although people can offer post-hoc reasons for their preferences.

Positioning analysis is facilitated by various mapping techniques that provide a visual representation of customer perceptions and/or preferences and how those drive choices. There are various mapping methods used in marketing. Mapping of perceptions vary depending on the nature of input data (e.g., data on how various offerings are similar or dissimilar to each other, or customers ratings or evaluations of various offering on different attributes). The various methods are summarized in Exhibits 1 and 2. We will describe three major approaches in greater detail: (1) perceptual maps from attribute-based data, (2) preference maps based on ideal points, and (3) joint-space maps.

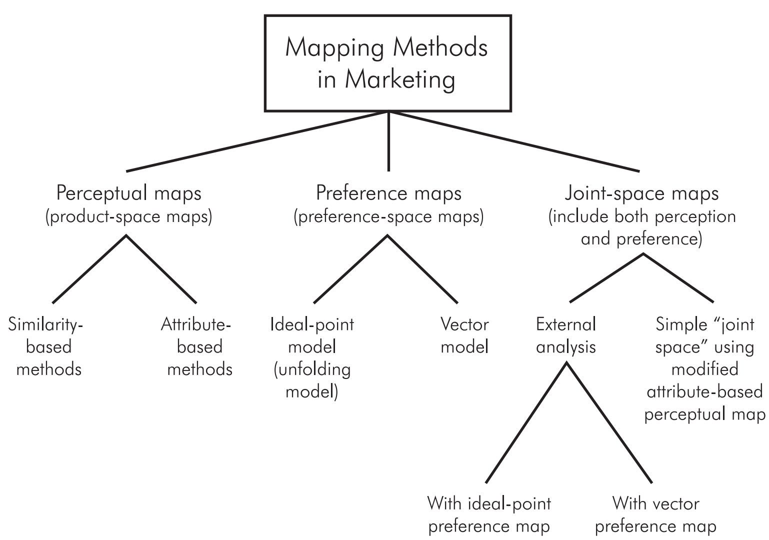


EXHIBIT 1

Mapping methods used in marketing fall into three categories: (1) perceptual maps, (2) preference maps, and (3) joint-space maps.

Model	Input	Output	Computer programs	Comments
Perceptual map from attribute-based methods	Data matrix consists of ratings of alternatives on a prespecified set of attributes. The data may either be from a single individual, or be averaged across members of a target segment.	Spatial map showing both the location of alternatives and the directional vectors associated with the attributes.	Factor Analysis (MDPREF) Discriminant Analysis	Particularly useful when market structure is driven primarily by tangible attributes, such as physical features of the product, its performance and its service characteristics.
Preference map from ideal-point methods (Unfolding model)	Data matrix consists of the stated or derived preferences of individuals over a set of alternatives. Preferences may be rank-orders (nonmetric unfolding model) or ratings (metric unfolding model).	Spatial map showing the locations of the alternatives and the ideal points of the individuals. An ideal point refers to an individual's most preferred combination of the underlying dimensions defining the product space and will vary from individual to individual.	KYST GENFOLD ALSCAL Quasi-Metric Unfolding	Particularly useful in categories where segment preferences are not uni-directional but exhibit an inverted U shape. For example, preference for coffee brands may decline as the available options become either "stronger" or "weaker" in flavor as compared to an individual's ideal brand.
Preference map from vector method	Data matrix consists of the stated or derived preferences of individuals over a set of alternatives, where preferences are measured on a rating scale.	Spatial map showing the locations of the alternatives and the directional vectors associated with the preferences of each individual.	MDPREF	Useful in categories where segment preferences are unidirectional along the underlying attributes. For example, if competition is driven by factors such as quality, value, or speed of response, preferences are likely to be increasing in a unidirectional manner.
Joint-space map from external analysis	Data matrix consists of the stated or derived preferences of individuals over a set of alternatives, supplemental with data giving locations of the alternatives from a perceptual map.	Spatial map showing the locations of the alternatives, the directions of the attributes, and the ideal points or the preference vectors of individuals.	Quasi-Metric Unfolding PREFMAP-3 GENFOLD	These are full-fledged systems with a number of processing options.
Simple joint-space map obtained from modified perceptual mapping methods	Data matrix may consist of (1) data like that in the similarity-based perceptual method where one of the alternatives included is a hypothetical ideal-point of the individuals or (2) data like that in the attribute-based perceptual method where one of the alternatives rated is the hypothetical ideal point of the individuals.	For data in (1), the spatial map provides the location of the alternatives, including the ideal alternative, and for data in (2), the spatial map provides the locations of all the alternatives, including the ideal alternative, and a directional vector for each attribute.	MDSCAL KYST INDSCAL MDPREF	These methods are fairly simple to implement, and are often insightful.

EXHIBIT 2

A summary of the major perceptual, preference, and joint-space modeling methods, their required inputs and outputs, and several computer programs that implement each method.

Description of Attribute-Based Perceptual Mapping

Perceptual mapping techniques offer a systematic method for extracting information about the underlying construct(s) from a data matrix consisting of customer perceptions on observable attributes. While there are several methods for doing this with attribute-based data, Hauser and Koppelman (1979) recommend factor analysis. We will describe the factor analysis procedure. The model we will use is called MDPREF, which contains options for a factor-analytic derivation of perceptual maps (Carroll 1972, and Green and Wind 1973).

Outline of the factor analysis procedure: Factor analysis is a technique for systematically finding underlying patterns and interrelationships among variables (here, attributes), based on a data matrix consisting of the values of the attributes for a number of different alternatives (brands, product classes, or other objects). In particular, it enables us to determine from the data whether the attributes can be grouped or condensed into a smaller set of underlying constructs without sacrificing much of the information contained in the data matrix. Factor analysis is also useful in preprocessing data before undertaking segmentation studies, as described in the appendix to this note.

Let X be a matrix with m rows and n columns, in which the column headings are attributes and the rows are alternatives, with the data in the matrix consisting of the average ratings of each alternative on each attribute by a sample of customers. Note that X is the transpose of the example data matrix for perceptual mapping shown in the previous subsection. Let X_s represent a standardized matrix in which each column of X has been standardized. (To standardize a column, for each value we subtract the mean of all values on that attribute and divide by the standard deviation of the values. By standardizing we remove the effect of the measurement scale and ensure that all variables are treated equally in the analysis—i.e., it would not matter whether income is measured in dollars or pesos.) We denote the columns of X_s as x_1, x_2, ..., x_n.

In the principal-components approach to factor analysis (the most commonly used method in marketing), we express each of the original attributes as a linear combination of a common set of factors, and in turn we express each factor also as a linear combination of attributes, where the jth factor can be represented as

$$F_j = a_{j1}x_1 + a_{j2}x_2 + \ldots + a_{jn}x_n \tag{1}$$

where the a's are weights derived by the procedure in such a way that the resulting factors F_j's are optimal. The optimality criterion is that the first factor should capture as much of the information in X_s as possible, the second factor should be orthogonal to the first factor and contain as much of the remaining information in X_s as possible, the third factor should be orthogonal to both the first and the second factors and contain as much as possible of the information in X_s that is not accounted for by the first two factors, and so forth.

Each value of the original data can also be approximated as a linear combination of the factors:

$$x_{kj} \approx z_{k1} f_{1j} + z_{k2} f_{2j} + \ldots + z_{kr} f_{rj}, \tag{2}$$

where the z_{kl}'s and f_{ij}'s are also outputs of the factor analysis procedure.

The relationships characterized by Eqs. (1) and (2) can be seen more clearly when represented as matrices (Exhibit 3). In Exhibit 3, the z's are called (standardized) factor scores and the f's are the factor loadings. Then Z_s is the matrix of standardized factor scores, and F is the factor loading matrix, with columns denoted as F_j, and those factor scores represent the correlation matrix of attributes with factors. (Note that the factors-by-attributes matrix in Exhibit 3 is actually the transpose of the F matrix.) If $r=n$, that is, if the number of factors is equal to the number of attributes, there is no data reduction. In that case, (2) becomes an exact equation (i.e., the approximation symbol in Exhibit 4.10, ≈, can be replaced by the equality symbol, =) that shows that the standardized data values (x_{kj}'s) can be exactly recovered from the derived factors. All that one would accomplish in that case is to redefine the original n attributes as n different factors, where each factor is a linear function of all the attributes. However, in perceptual mapping we seek r factors (r typically being 2 or 3) that retain as much of the information contained in the original data matrix as is possible. Variance (the dispersion of values around a mean) is a measure of the information content of an attribute. The larger the variance, the higher the information content. Once we standardize the attributes, each attribute contains one unit of variance (except for attributes for which all values are identical, in which case the information content of that attribute is equal to 0). If there are n attributes in the analysis, then the total variance to be explained (information content) is equal to n.

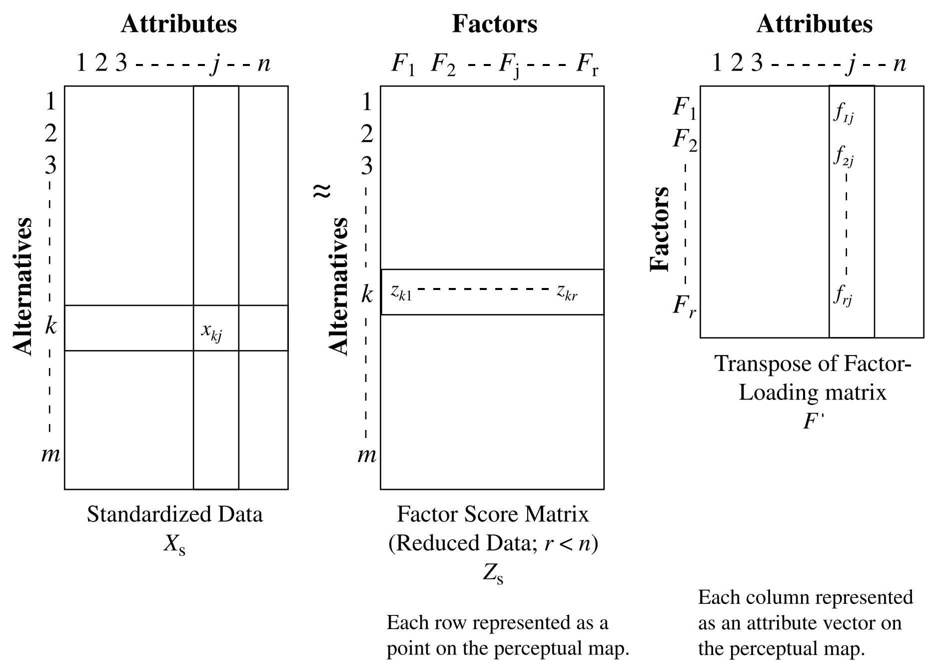


EXHIBIT 3

A pictorial depiction of attribute-based perceptual mapping. The model decomposes the (standardized) original data matrix (X_s) as a product of two matrices: (1) the standardized factor score (Z_s) matrix and (2) the factor-loading matrix (F); r is the number of factors (dimensions of the perceptual map) and is usually set to be equal to 2 or 3.

The output of a factor analysis procedure is illustrated graphically in Exhibit 4 for the case of two attributes, elegance and distinctiveness, in a study of notebook computers. The procedure first finds a factor along which the points are maximally dispersed (i.e., this factor has the maximal variance when we project the points onto it). In this example the locations of the notebook computers are dispersed much more along factor 1 than factor 2. If factor 1 has a variance equal to 1.7, this factor alone accounts for 85 percent of the information content in the two attributes ([1.7 / 2.0]×100), suggesting that "elegance" and "distinctiveness" are correlated and possibly refer to a common underlying dimension called "design." The procedure then finds a second factor, orthogonal (perpendicular) to the first, which maximally recovers the remaining variance. In this case the remaining factor will recover 15 percent of the variance; together the two factors explain all the variance in the data. If there are n attributes, the procedure continues in this fashion until it extracts as many factors (up to n), all orthogonal to each other, as are needed to explain the variance in the original data.

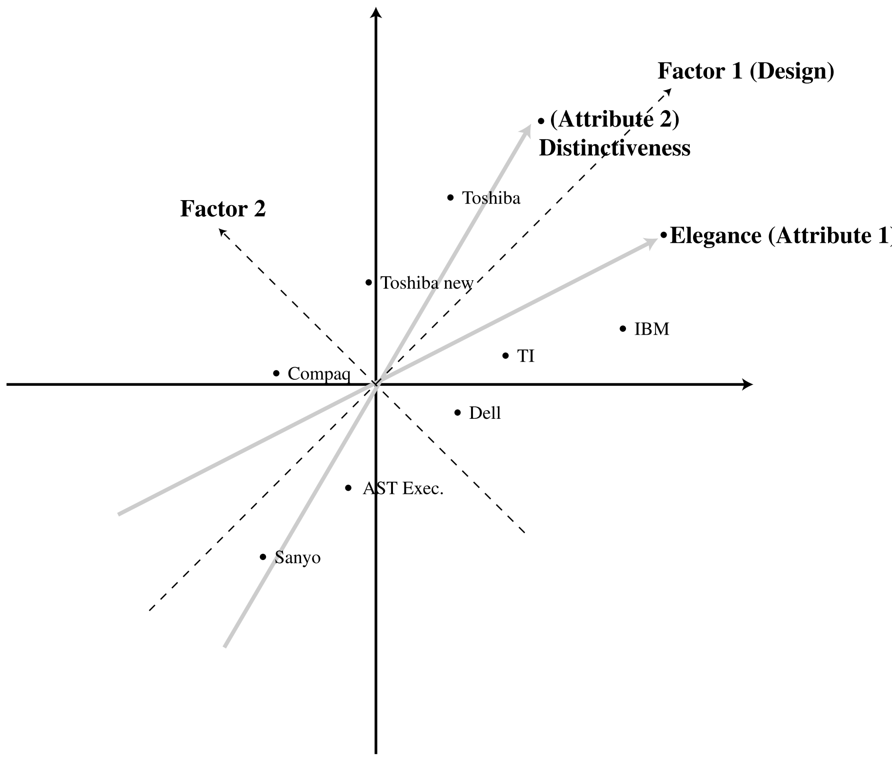


EXHIBIT 4

A two-attribute example of factor analysis for notebook computers. “Distinctiveness” and “elegance” are correlated with each other, and they are represented by an underlying factor (dimension) called “design.” For this example a one-dimensional map captures most of the variation among the notebook computers.

Interpreting factor analysis output. An important objective of factor analysis is to provide an interpretation of the underlying factors in terms of the original attributes. The key to interpretation is the factor-loading matrix F. By looking at the pattern of the loadings, we should be able to identify and name the factor. Loadings that have high absolute value (high absolute values of correlations) make interpretation easy. In a perceptual map the factor-loading matrix is represented visually as attribute vectors, where correlation between any attribute and a factor is equal to the cosine of the angle between that attribute vector and the corresponding factor.

The factors may be rigidly rotated (i.e., F is transformed by an orthogonal matrix, while at the same time making the corresponding transformation to Z_s) to aid interpretation, forcing attributes to have either big or small cosines with the transformed factors (the transformation is called Varimax rotation). The result is that a set of attributes tends to line up closely with each factor. In this way, attributes tend to be closely aligned with a single factor. We can then better identify the attributes most closely associated with the transformed factors. Although rotation changes the variance explained by each factor, it does not affect the total variance explained by the set of retained factors. *To further aid interpretation, we can draw each attribute vector on the map with a length that is proportional to the variance of that attribute explained by the retained factors*. Exhibit 5 is a perceptual map derived from factor analysis, where the length of each attribute vector indicates the proportion of the variance of that attribute recovered by the map.

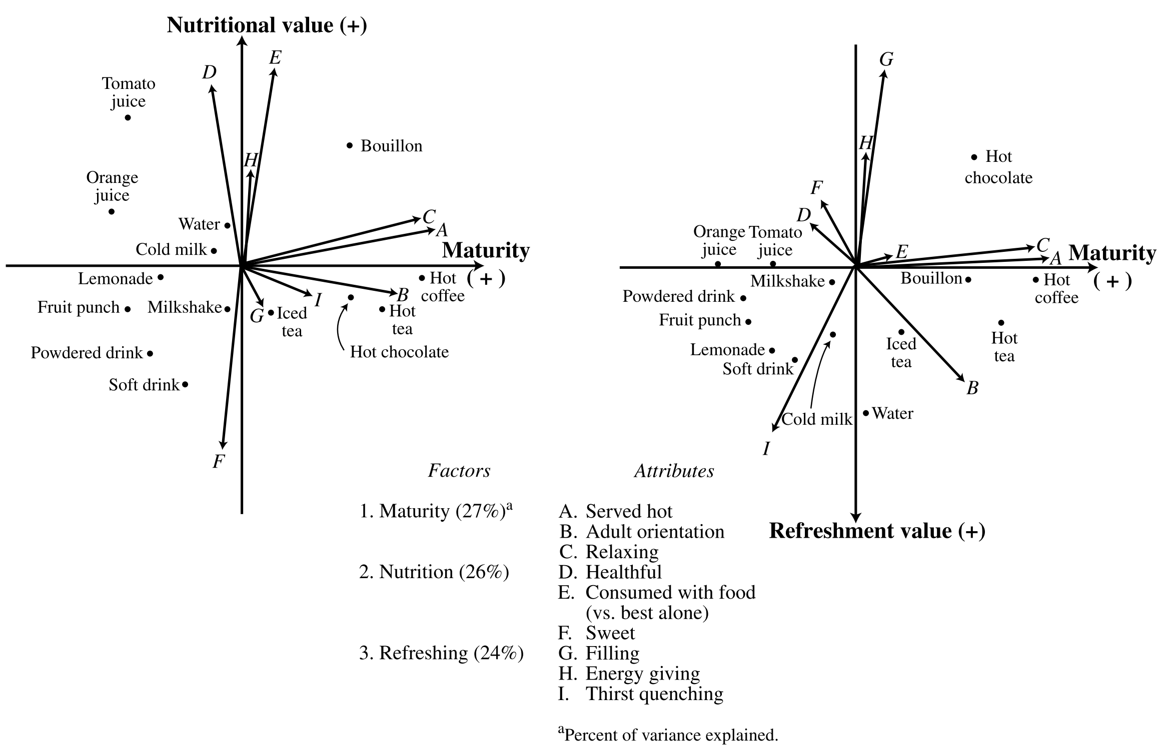


EXHIBIT 5

An example of a three-dimensional attribute-based perceptual map of beverages, where the length of each attribute vector is proportional to the amount of its variance explained by the map. The three dimensions are (1) maturity of target segment, (2) refreshment value, and (3) nutritional value. *Source*: Aaker and Day 1990, p. 574.

Variance explained by a factor: Each factor explains a proportion of the total variance in the data as follows:

$$\text{Variance explained by factor } i = f_{i1}^2 + f_{i2}^2 + \ldots + f_{in}^2 \qquad (3)$$

The proportion of the variance explained by a single factor equals the variance explained by that factor divided by n, the total variance in the data. In Exhibit 5 the proportion of variance explained by the horizontal axis (factor 1) is equal to 0.27, and the variance explained by the vertical axis (factor 2) is equal to 0.26, giving a combined variance explained by the two axes of 0.53. If all n factors are retained, these proportions will sum to one.

Proportion of an attribute's variance explained by the retained factors: A good factor analysis solution explains a significant proportion of the variance associated with each original attribute as follows:

$$\text{Proportion of variance explained for attribute } j = f_{1j}^2 + f_{2j}^2 + \ldots + f_{rj}^2 \quad (4)$$

Number of factors retained: If the variance of any attribute is poorly recovered by the retained factors, that attribute is unique and would require additional factor(s) for it to be explained. In that case we might consider going to a higher-dimensional map, say, from a two- to a three-dimensional map. This raises the broader question of how many factors we should retain in a factor-analysis study. Unfortunately, there is no simple answer to this question, although there are several useful guidelines. In the context of perceptual maps, it rarely makes sense to go beyond three dimensions, especially if a three-dimensional map recovers more than 60 to 70 percent of the variance in the original data. One useful guideline is that every retained factor should individually account for *at least* one unit of variance (equivalent to the variance in a single attribute) and typically should account for substantially more than one unit of variance.

Location of the products (alternatives) on a perceptual map: An important element of the factor analysis output is the factor score matrix, It gives the location of each product on each factor. If we retain only two factors, then the location of the first product in a two-dimensional perceptual map is given by the first two elements in the first row of the factor-score matrix; the location of the second product is given by the first two elements of the second row of the factor score matrix, and so on.

In summary, attribute-based methods provide a powerful set of tools for perceptual mapping. They are particularly useful when the product alternatives are differentiated along tangible attributes that are well understood and evaluated by customers.

Description of Ideal-Point Based Preference Mapping

We briefly describe a geometric model which is technically referred to as Multidimensional Unfolding. The basic idea is to represent customers and products as points on a map so that products that are closer on the map to a customer point are preferred to a greater extent by that customer as compared to products that are farther away from that customer point. In other words, the predicted preference of a product to a customer is inversely related to the distance between that consumer customer point and the product point on the map. The locations of customers on the map are referred to as "Ideal Points" because a product located exactly at a customer point has zero distance from that customer's location and, therefore, will be the product that is preferred by that customer over all other products. In other words, a product located exactly where the customer is located on the map is the "ideal" product for that customer.

Quasi-metric approach to preference mapping. Many methods have been proposed for generating preference maps with ideal points. Here, we briefly describe the quasi-metric approach developed by Kim, DeSarbo and Rangaswamy (1999). We restrict ourselves to the case where customer preference data for various products are obtained on a rating scale (say, a 100-point scale) where higher numbers indicate greater preference for the product.

Let:

$s_{ij} \in S$: preference rating of customer *i* for product *j*.

$x_{ik} \in X_i$: customer i's ideal point coordinate on dimension k of the map

$y_{jk} \in Y_j$: product j's coordinate on dimension k of the map

$d_{ij} \in D$: the Euclidean distance between customer (ideal point) *i* and product point *j* in a given K dimensional joint-space map (usually, K is at most equal to 3)

$e_{i(j,j')n} \in E$: a set of variables representing the preference differentials for customer *i* between the n^{th} pair of products, when the stimuli are arranged in decreasing order of preference according to the resulting map. That is, $e_{i(j,j')n} = d_{ij'} - d_{ij}$. There are (J-1) ordered pairs among the J ordered stimuli for each subject i. That is, $n \in N$, where N is an index set from 1 to J-1.

The distance between ideal point i and product j in a K-dimensional joint-space is defined by:

$$d_{ij} = [(X_i - Y_j)(X_i - Y_j)']^{1/2} \qquad \forall i \in I, \forall j \in J. \qquad (5)$$

Then, we approximate an unknown monotone relationship between stimuli j and j' for customer i in a K-dimensional Euclidean space where:

$$s_{ij} \le s_{ij'} \Rightarrow d_{ij} \le d_{ij'}, \qquad \forall i \in I, \forall j, j' \in J. \qquad (6)$$

The restrictions in (6) ensure that the closer a product is positioned to an ideal point in the preference map, the more it will be preferred by that customer. Thus, the preference differential between the n^{th} ordered pair of products j and j' for customer i, $e_{i(j,j')n}$, can be defined and represented as:

$$e_{i(j,j')n} = d_{ij'} - d_{ij}, \qquad \forall i \in I, \forall j, j' \in J, \qquad (7)$$

where product j is at least as preferred as products j' for customer i. Without any loss of generality, we can replace (6) with the following condition:

$$e_{i(j,j')n} = d_{ij'} - d_{ij} \ge 0.0 \Leftrightarrow s_{ij} \le s_{ij'}, \qquad \forall i \in I, \forall j, j' \in J, \qquad (8)$$

Monotonicity assures that more preferred products for any customer are at least as close to the ideal point of that customer as compared to a less preferred product. This condition implies that all preference differentials for each customer i ($e_{i(j,j')n}$) should be non-negative.

To determine the unknown locations of the ideal points and product points, the quasi-metric preference mapping procedure first transforms the raw data of preferences into an *a priori* matrix of desired distances between the ideal points and the product points that have to be satisfied by a preference map. This matrix is denoted as Λ. In general, the desired distances embedded in Λ cannot be fully satisfied by preference maps in a low-dimension (say 2 or 3 dimensions).

Therefore the solution procedure attempts to satisfy the constraints in Λ as closely as possible by using an alternating, two-stage process to find a map in which the distances between the points are as close to being proportional to Λ as possible. This procedure starts from an initial configuration (D) that results in a set of distances between all the points (both ideal points and product points). Some of the distances in D will be well correlated to the corresponding distances in Λ while others will be poorly correlated. In the first stage of the solution procedure, this discrepancy information is used to construct a new set of "target distances" denoted as $\widetilde{D}_i$, which departs optimally from Λ (in the sense of least squares). In the second stage, this set of target distances is used to generate a new configuration of points that minimizes a normalized loss function which increases as the distances in the generated configuration depart from being equal to the corresponding distances in $\widetilde{D}_i$. The procedure alternates between the two stages until no further significant reduction in the loss function is achieved between the two stages. Further details about this procedure are available in Kim, DeSarbo and Rangaswamy (1999).

To determine the appropriate number of dimensions for the preference map, we trade off between monotonicity and the number of dimensions. The general idea is to start with one dimension, and increase the number of dimensions until the loss function decreases very little with the addition of each incremental dimension.

Interpreting ideal point preference maps: The interpretation of a preference map is usually straightforward. As output, the software produces points representing products and customers. Any natural groupings of ideal points on the map denote potential customer sub-segments. For example, in Exhibit 6, the circles represent segments, and the size of a circle represents segment size. These customers prefer Budweiser the most (i.e., their ideal points are located closest to Budweiser), Miller the next most, and so forth. The next largest segment is marked as a circle with the number 2. This segment prefers Coors Light and Michelob most. The model also shows that Stroh's is not the most preferred brand in any segment—it is a "compromise brand" that some respondents may choose (e.g., those in segments 5 and 3) when their most preferred brand is unavailable.

To fully understand and use a preference map, we also need to interpret the axes, which may sometimes be difficult. One simple way to interpret the axes is to look for alternatives at extreme locations on each dimension and then try to determine the differentiating features between these alternatives. We can also shift the origin or rotate the axes to improve interpretation. Usually, however, we will need additional data from the study participants regarding their perceptions of the alternatives to help us interpret the map. To incorporate such data in preference maps, we need to generate "joint-space": maps described next.

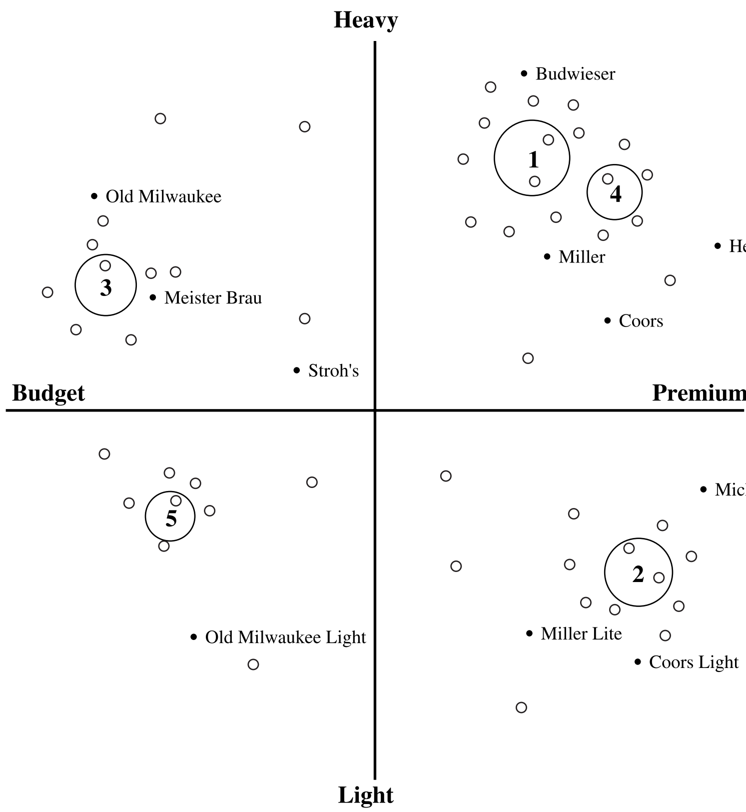


EXHIBIT 6

A joint-space map derived from external analysis with groupings of customer ideal points in circles. The size of the circle indicates relative size of the customer segment at that location. *Source*: Adapted from Moore and Pessemier 1993, p. 146.

Description of Joint Space Mapping

A major limitation of perceptual maps is that they do not indicate which areas (positions) of the map are desirable to the target segments of customers and which ones are not. In other words, the maps do not incorporate information about customer preferences or choices. A major limitation of preference maps is that they do not tell us what product attributes should be changed to make a product more attractive to the target customers. We need to use *joint-space mapping methods* to incorporate both perceptions and preferences in the same map.

Perceptions are fundamentally different from preferences: customers may see Volvo as the safest car, but they may also have a low preference for it. In addition, unlike perceptions preferences do not necessarily increase or decrease monotonically with increases in the magnitude of an attribute. In some cases (e.g., sweetness of soft drink) each customer has an ideal level of the attribute above or below which a product becomes less preferred. In other cases customers always prefer more of the attribute (e.g., quality of a TV set) or always prefer less of an attribute (e.g., waiting time before a car is repaired). Exhibit 7 illustrates these different types of preferences. Preference maps that incorporate inverted U-shaped preferences are referred to as ideal-point (or unfolding) models. Maps that incorporate linear preference functions are referred to as vector models. (In a third kind of preference modeling, we can use part-worths to represent arbitrary piecewise linear functions that can approximate both ideal-point and vector preference functions. A method called Conjoint Analysis is useful for this purpose.

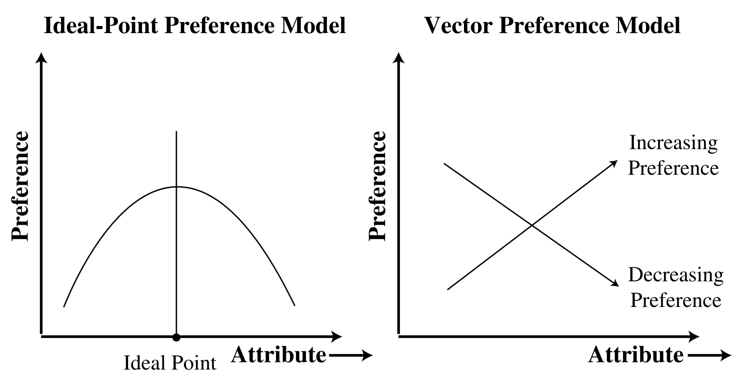
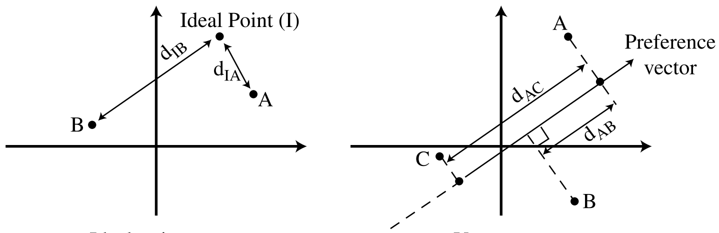


EXHIBIT 7

Different types of preference functions. Ideal-point models have an intermediate “best level,” e.g., sweetness, whereas for vector models more (or less) is always more (less) preferred, e.g., waiting time, reliability.



Ideal-point map
A is preferred twice as much as B
($d_{IB} = 2d_{IA}$)

(a)

Vector map
A is preferred to B and
B is preferred to C.
With reference to A, C
is preferred half as much as B
($d_{AC} = 2d_{AB}$)
(b)

EXHIBIT 8

Interpreting simple joint-space maps. In ideal-point maps distances directly indicate preference: the larger the distance from the ideal point, the less preferred the brand. In vector maps the product locations are projected onto a preference vector (dashed lines in b), and distances are measured along the preference vector.

Simple joint-space maps: The simplest way to incorporate preferences in a map is to introduce a hypothetical ideal brand into the set of alternatives that customers evaluate in the attribute-based perceptual mapping model. For each respondent, an ideal brand has that individual's most preferred combination of attributes. Assuming that both the perceptions and the preferences of customers in a target segment are fairly homogeneous, we can find the location of the "average" ideal brand using either similarity-based or attribute-based methods. The ideal brand thus becomes simply another alternative that customers evaluate. In the resulting map, locations that are farther away from the ideal point (location of the ideal brand) are less desirable to customers than locations closer to the ideal point. Using this approach in Exhibit 8(a), we can regard alternative A, which is twice as far from the ideal point as alternative B, as being preferred half as much as B.

Another way to include preferences in attribute-based models is to add an attribute called "preference" on which customers rate all the alternatives to indicate their preferences for these alternatives. When we aggregate and average these preference ratings, we can treat the average ratings as an additional row in the input data matrix to represent an attribute called "preference." Alternatively, if we have the current market shares for the various products being mapped, we could use those market shares as surrogate indicators of preferences for those products. The map we generate from this modified data set then includes a preference vector to indicate the direction of increasing preference. An alternative positioned farther along this vector is one for which customers have greater preference. Suppose that alternative A is farthest along the preference vector. Then if B is half as far from A as C is from A along the preference vector, customers prefer B twice as much as C (Exhibit 8b).

Exhibit 9 shows a simple joint space map of notebook computers derived by using the above approach. The preference vector shows that customer preference increases with improvements in screen quality and perceived value of the product and decreases with lower levels of battery life. In this example the two-dimensional map recovered over 80 percent of the variance in the preference "attribute." However, if it had recovered a low percentage, say less than 50 percent, then it would be unwise to use the map to interpret preference structure, even though the map could still be useful for interpreting the perceptual dimensions. When variance recovery for the preference vector is poor, it may be worthwhile to drop some attributes from the analysis to see if you can produce a joint-space map that is easier to interpret.

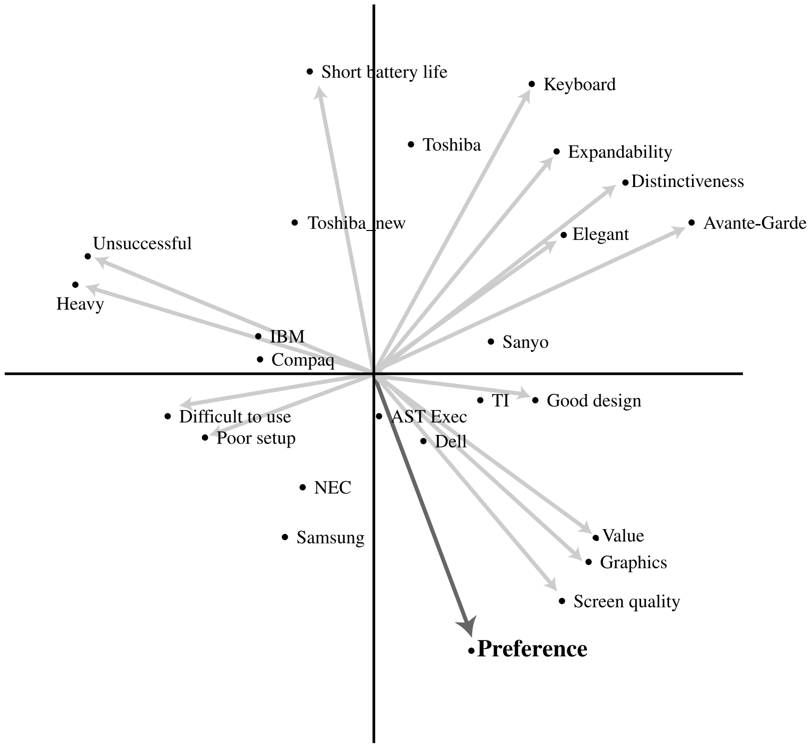


EXHIBIT 9

In this example of a simple attribute-based joint-space map with a preference vector, the direction of increasing preference is indicated by the attribute "preference." Overall preference for notebook computers increases with screen quality, value, and long battery life but is unaffected by expandability, keyboard, and ease of use.

Joint space mapping using external analysis: An external analysis mapping procedure first creates a map of all the products using attribute-based perceptual mapping. The mapping model is based on the assumption that respondents who have common perceptions of a set of alternatives may have widely differing preferences for these alternatives. Ideally, the underlying perceptual map should be derived from the same set of respondents from whom preference data is obtained, but this is not crucial.

The "external" perceptual map serves to fix the relative positions of the different competing products. Customer preferences are then superimposed onto the perceptual map so that each customer's stated preference ratings are recovered as closely as possible in the resulting joint-space map. This can be done in one of two ways. (1) In the ideal point version, customer locations are superimposed on the perceptual map in a manner that best captures the relative preferences a customer has for each product, i.e., the ideal point location for a customer is chosen so that the distances of that ideal point from each of the product points recovers as closely as possible the relative preferences of that customer for those products. (2) In the preference vector version, each customer's direction of increasing preference is chosen so that it recovers that customer's preference orderings of the products as closely as possible. In what follows, we briefly elaborate on both these approaches.

Joint-space mapping with ideal points: Here the quasi-metric unfolding model of Kim, DeSarbo and Rangaswamy (1999) is modified so that every configuration considered by the solution procedure fixes the ***relative*** locations of all the product points (y_{ik}) to be identical to those determined by the external perceptual map. The solution approach is identical to what we had outlined earlier for the preference map version of the quasi-metric model.

Joint-space mapping with preference vectors: Here, we introduce for each respondent a preference vector into the map in a manner that ensures maximal correspondence between the input preference ratings (or rankings) for the alternatives and the preference relationships among the alternatives in the resulting joint-space map. Each customer included in the study has a unique preference vector.

Let s_{ij} denote the value of the preference rating of the jth alternative by the ith customer. The solution procedure attempts to find a preference vector (i.e., the direction in which preference increases) for each customer by using the following equation to compute estimated ratings:

$$\hat{s}_{ij} = a_i \sum_{k=1}^{r} x_{ik} y_{jk} + b_i, \qquad (9)$$

where

a_i = slope of the preference vector;

b_i = intercept term for preference vector;

y_{jk} = coordinate location of alternative j on dimension k, determined from a perceptual map;

x_{ik} = preference vector coordinate on dimension k; and

r = number of dimensions in the perceptual map.

Given r and y_{jk} (for all j and k), the model attempts to find a_i, b_i, and x_{ik} such that $\hat{s}_{ij}$ is as close as possible (in the sense of minimizing squared distance) to the ratings s_{ij}. (To draw the map, we can relocate the computed preference vectors by shifting them in a parallel manner so that they pass through the origin.) The product term, $x_{ik}\ y_{jk}$, in Eq. (9) ensures that the preference vector direction on the map will maximally recover the preference ratings s_{ij} for respondent i for all j, for the given positions of the product alternatives (y_{jk}, j=1, 2, ... , J and k=1, ... , r). Further details about the PREFMAP3 model for implementing this procedure are provided in Carroll (1972), and Green and Wind (1973), and Meulman, Heiser, and Carroll (1986).

Exhibit 10 shows a map derived using the preference vector developed by PREFMAP-3, which is equivalent to the "Ideal Point" map shown in Exhibit 6. To interpret the preference vectors, follow the guidelines we gave for interpreting attributes in perceptual maps.

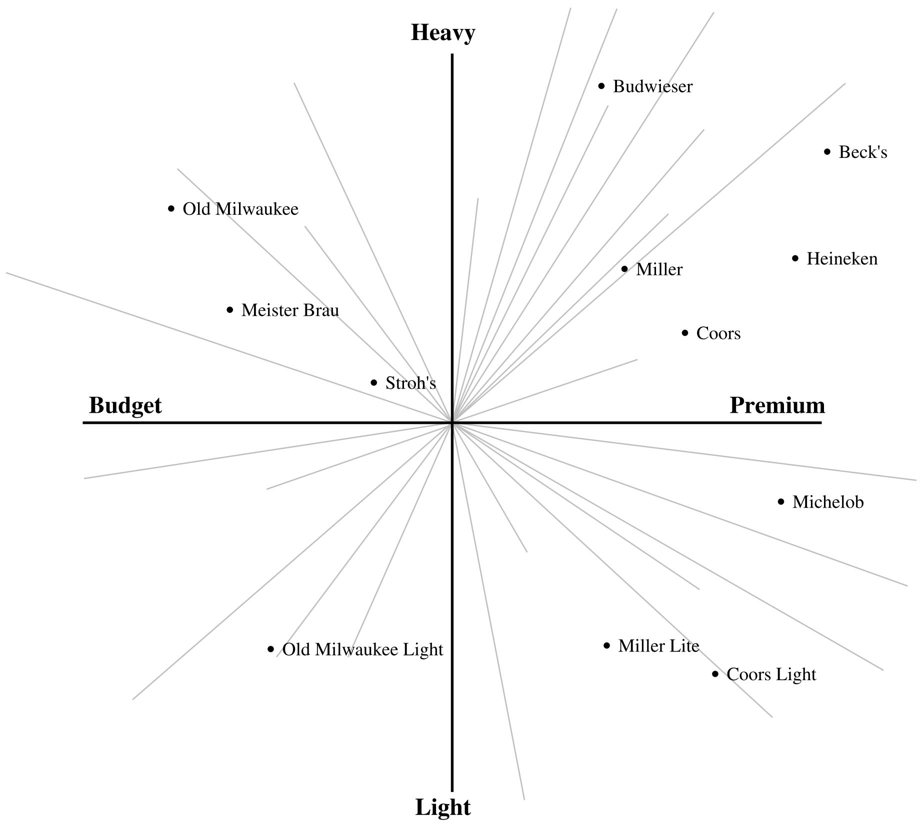


EXHIBIT 10

A joint-space map derived from external analysis displaying the preference vectors of 25 respondents. This map is equivalent to the map shown in Exhibit 6, which was based on the ideal-point version of external analysis. The lines are denser in the upper right quadrant, suggesting that more respondents prefer the brands in that quadrant. The length of a preference vector indicates the degree to which the map was able to capture the preferences of that respondent.

Transforming Preferences into Choice Shares or Market Shares

Preference data embedded either in either the standalone preference maps or in joint-space maps may also be used to compute an index of predicted choice shares (or, equivalently, market shares if customer's budget constraint Is unlikely to be a major factor in the context under study) ***for any of the products included in the study at any location on the map***. Such predictions are useful for exploring strategic options to re-position a focal product or brand. We can consider two “choice rules” for computing choice shares: (1) first choice and (2) share of preference. Under the first choice rule, we assume that each customer only purchases the most preferred product (that is, the one closest to the ideal point or the one farthest along a preference vector). Under the share of preference rule, we assume that each customer purchases every product in some proportion to its measured preference value (relative to the sum of the preference values for all other products included in the model). The first choice rule is appropriate for infrequently purchased products (e.g., cars), whereas the share of preference rule is appropriate for frequently purchased products (e.g., shampoo or soft drinks). The software can automatically do these computations and enable us to explore the potential market performance that can be achieved by repositioning any one product assuming that all other products remain at their original locations on the map.

To compute choice share at any location on the map, we have two choice rules we could use: (1) First choice (also called maximum utility) rule, and (2) Share of preference rule. In what follows, we describe the methods we could use to compute these shares. It is best to interpret the computed choice shares as indicating *relative attractiveness* of the selected location on the map for the selected product (relative to choice share computed at other locations, such as the current position of a brand on the map), rather than as an indicator of the actual choice share that will be realized in the marketplace.

Choice shares computed from ideal-point model: First, we create a matrix containing the pairwise distances between an ideal point (location of person) and the location of the products. As an example, consider the following points on a preference map (or a joint-space map) with two customers (c1 & c2) and two products (p1 and p2) along three dimensions:

	dim1	dim2	dim3
c1	1	2	3
c2	2	1	2
p1	1	3	3
p2	0	1	0

Then the distance matrix is computed as follows:

Distance between c1 and p1 = $\sqrt{(1-1)^2 + (2-3)^2 + (3-3)^2}$ = 1.0

Distance between c1 and p2 = $\sqrt{(1-0)^2 + (2-1)^2 + (3-0)^2}$ =3.32

The distance matrix is then given by:

	p1	p2
c1	1.0	3.32
c2	2.0	2.83

To compute choice share under the first-choice rule, we first determine the number of customers for whom product 1 is the closest (has the smallest distance) and the number of customers for whom product 2 is closest. Market share for the product j is then equal to:

$$\frac{\text{Number of customers for whom the product j is located closest to their ideal points}}{\text{Total number of customers}}$$

In our two-product example, product 1 is closest to both customers ideal points (as compared to product 2) and, therefore, its market share will be 100% (assuming the market consists of 2 customers), and market share for product 2 will be 0.

To compute choice share under the share of preference rule, we first compute a preference scale value for each customer for each product as the inverse of the distance of a product from the ideal point. Then choice share for product j for any customer i given by:

$$Share_{ij} = \frac{\text{Inverse of distance (i.e., 1/X) for product j for customer i}}{\text{Sum of inverse distances across all products for customer i}}$$

Then the choice share for product j considering the entire market is computed by adding the share of product j for each of customer in the study, divided by the total number of customers:

$$\frac{\sum_{i} \text{Share}_{ij}}{\text{Total number of customers in the study}}$$

For the above example, share for product 1 will be:

$$= \frac{\dfrac{1}{1+\dfrac{1}{3.32}} + \dfrac{\dfrac{1}{2}}{\dfrac{1}{2}+\dfrac{1}{2.83}}}{2} = \frac{0.768+0.586}{2} = 0.677$$

Likewise, choice share can be computed for 2.

Choice shares computed from preference vector model: For each customer, first project each product on to that customer's preference vector. We also normalize each customer's preference vector to be of unit magnitude:

	dim2	dim2	dim 3
c1	0.267	0.535	0.802
c2	0.667	0.333	0.667
p1	1	3	3
p2	0	1	0

Then the projection of each product onto the customer's preference vector is given by:

Projection of p1 on c1 = 1*0.267+3*0.535+3*0.802 = 4.278
Projection of p2 on c2 = 0*0.267+1*0.535+0*0.802 = 0.535

Likewise,
Projection of p1 on c2 = 3.667
Projection of p2 on c2 = 0.667

To compute choice share under the first-choice rule, we first determine the number of customers for whom product j has the highest projected preference score. Market share for the product j is then equal to:

$$\frac{\text{Number of customers for whom product j has the highest projected preference score}}{\text{Total number of customers}}$$

For this example, product 1 has the highest preference score for each customer (4.278 versus 0.535 for customer 1, and 3.667 versus 0.667 for customer 2). Product 1's market share will be 100% (assuming the market consists of 2 customers), and market share for product 2 will be 0.

Let PP_{ij} be the projected preference score of product j for customer i. Under the share of preference rule, choice share for product j for any customer i can be computed as:

$$Share_{ij} = \frac{e^{PP_{ij}}}{\sum_j e^{PP_{ij}}}$$

The exponentiation of the projected preference score ensures that the computed market share of every product lies between 0 and 1 and the share formula given above ensures that the sum of choice shares across products will equal 1 for each customer. Then the choice share for product j considering the entire market is computed by adding the share of product j for each customer in the study, divided by the total number of customers:

$$\frac{\sum_i \text{Share}_{ij}}{\text{Total number of customers in the study}}$$

For our example, share for product 1 will be:

$$= \frac{\dfrac{e^{4.278}}{e^{4.278} + e^{0.535}} + \dfrac{e^{3.667}}{e^{3.667} + e^{0.667}}}{2} = \frac{0.977 + 0.953}{2} = 0.965$$

Note: In our examples, the preferences of the two customers, c1 and c2, as represented by the ideal points and preference vectors are not directly comparable, which is why the market shares as computed by the share of preference model for the ideal point version and the preference vector version are different.

Incorporating Price in Perceptual Maps

W can represent price in several ways in perceptual maps as described in chapter 4 of Lilien, Rangaswamy and De Bruyn (2007). In attribute-based perceptual maps, we can include price as another attribute along which customers evaluate all the products. Or we can include objective prices (the actual prices of the products) as an additional attribute in developing the map. Another way to approach this issue is to divide the coordinates of each alternative by its price along each of the dimensions of the map.

Summary

Mapping techniques enable managers to understand the competitive structure of their markets. Based on this understanding, they can then position their offerings to gain a favorable response from their target segments. Although these techniques are powerful, it is important to understand their limitations so that they we can apply them where they are most useful. The insights provided by perceptual maps are limited by the particular set of alternatives and attributes included in the study—that is, they support positioning efforts within an existing framework. The insights provided by preference maps and joint-space maps (e.g., predicted market shares associated with a re-positioning option) are relevant only for the target segments participating in the study. We should also remember that the mapping techniques only serve to *represent* perceptions and preferences in a manner that aids decision making. They do not tell us much about *why* customers form certain perceptions or preferences.

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